

1. Solve $\frac{dy}{dx} + \frac{y}{x} = 3x$.

Soln The given equation is

$$\frac{dy}{dx} + \frac{y}{x} = 3x. \quad \text{--- (1)}$$

It is of the form $\frac{dy}{dx} + Py = Q$.

Here, $P =$ coefficient of y in (1) $= \frac{1}{x}$

$$Q = 3x.$$

$$\therefore \text{IF} = e^{\int P dx} = e^{\int \frac{1}{x} dx} = e^{\log x} = x.$$

Multiplying (1) by IF, we have

$$x \left(\frac{dy}{dx} + \frac{y}{x} \right) = 3x^2$$

$$\Rightarrow x \frac{dy}{dx} + y = 3x^2$$

$$\Rightarrow x dy + y dx = 3x^2 dx$$

$$\Rightarrow d(xy) = 3x^2 dx$$

Integrating, we get

$$\Rightarrow xy = 3 \cdot \frac{x^3}{3} + K$$

$$\Rightarrow xy = x^3 + K$$
